Why Do Older People Quit The Stock Market?*

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Abstract

We document that in the U.S., the stock market participation rate over the life cycle decreases as people get older. This fact can not be captured by standard model where smooth expected utility function always allows decision

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maker stay in the stock market given positive equity premium and independence between older people's non-asset income and stock return. To explain this puzzle, we introduce Knightian uncertainty in a multi-prior utility model where agents have ambiguity towards the correlation between risky stock return and uncertain health expenditure. In this environment, older people quit the stock market under some range of ambiguity towards the correlation. Within this range, they do not long stocks since they worry stocks are too much like the non-asset income. Similarly, they do not short sell stocks because they also worry that stocks and their non-asset income may co-move very negatively.

Keywords: Portfolio Choice, stock market participation, ambiguity

JEL classification: G11, E21, I19

1 Introduction

It has been well documented that a significant proportion of households in the U.S. do not hold stocks directly or indirectly (for example through mutual funds or retirement accounts). However, less attention has been paid to stock market participation behavior over the life cycle. We observe from SCF and PSID data that the stock market participation rate decreases when people get older. We first formulate this data pattern as a puzzle and try to answer the question that why the older people quit the stock market.

We find this decreasing stock market participation pattern puzzling. In a standard life cycle portfolio choice model, when the expected utility vNM function is locally risk neutral or even as Segal and Spivak (1990) point out, is of first order risk aversion, as long as the expected equity premium is positive and labor income is independent to stock return, the agent will always put positive amount investment in stock market. Since this puzzle is a qualitative one, we think it is even "deeper" than the famous equity premium puzzle (Mehra and Prescott (1985)) since the latter is a quantitative puzzle.

When we consider the investment decisions made by aged people, the environment they are facing is exactly what we described above. Their nonfinancial income, largely from social security benefits, are fixed or at least no more correlated with the stock return than when they're young. If they already overcome the fixed entry and/or transaction costs of stock investment which a major stream of literature claim to be the reason that people defer the participation, according to the standard theory, they should stay in the market rather than quitting. So why quit?

Rosen and Wu (2004) find that among aged people, there is a significant correlation between stock market participation and health status. Motivated by their finding, we believe that the health shock that older people faces is the key to solve this life cycle stock market participation puzzle. We model health shock as a health expenditure risk which affects the de facto nonfinancial income that aged people receive. Young people only face risky stock return, while older people face a joint distribution between risky stock return and risky nonfinancial income due to health shock.

Deviating from the standard expected utility framework, we then introduce ambiguity towards correlation between stock return and de facto nonfinancial income into a multi-prior utility model as in Gilboa and Schmeidler (1989). Building on the work of Dow and Werlang (1992), Epstein and Schneider (2003) and Cao, Wang and Zhang (2005), we show that limited participation may arise as an equilibrium result with heterogeneous uncertainty-averse agents when people are getting older.

In our model, older people are uncertain about the correlation between stock return and de facto nonfinancial income in their joint distribution and they are heterogeneous in their levels of uncertainty. We show that there exists a range of ambiguity level over which investors neither hold long or short sell a stock. Heterogeneity on the ambiguity guarantees some people will fall into this range hence they will choose not to participate in the stock market.

The reason why some agents would not hold long position of a stock is agents worry that the correlation is sufficiently positive so that nonfinancial income is very much like stock. Think about the following scenario: agents might face a bad health shock which significantly reduces their nonfinancial income. However, the correlation between stock return and de facto nonfinancial income is high so that stock return might go down too. Agents thus cannot cover their medical expenditure by the investment. On the other hand, some agents worry that the correlation is sufficiently negative so that nonfinancial income is very much dislike stock. Again think about the following scenario: suppose they short sell a stock today, tomorrow when they get sick, nonfinancial income decreases, but it is also the time the stock price is higher and they have to purchase the stock back to cover the position. Financial loss thus is incurred. Therefore, ambiguity from both extreme cases prevents older people from holding either long or short position in stocks.

Our work contributes to the limited participation literature such as Mankiw and Zeldes (1991), Vissing-Jørgensen (2002a, 2002b), and Haliassos and Bertaut (1995). Mankiw and Zeldes (1991) and Vissing-Jørgensen (2002a, 2002b) emphasize the importance of difference between stock holders and nonstockholders in explaining equity premium puzzle and estimating the elasticity of intertemporal substitution (EIS). In their papers, they treat stock market participation as exogenously given, while in our model, limited participation is an equilibrium result. They also only look at the cross-section stock market participation, not the life cycle phenomenon as we focus on. Haliassos and Bertaut (1995) explicitly point out that limited stock market participation is a theoretical puzzle from the viewpoint of expected utility model. They investigate empirically a number of potential explanations for this puzzle. They find the degree of risk aversion, heterogeneity of beliefs, habit persistence, time non-separability, and borrowing constraint, which usually help to explain the equity premium puzzle, do not account for the phenomenon. This confirms our claim that stock participation puzzle is a deeper one than equity premium puzzle. They show that inertia and departures from expected utility framework might be a promising explanation. Our work, based on a model with ambiguity towards correlation between stock return and de facto nonfinancial income in a multi-prior utility framework, can be viewed as a theoretical confirmation of their claim.

Our work is also related to the literature on portfolio choice over the life cycle such as Jagannathan and Kocherlakota (1996), Cocco, Gomes and Maenhout (2005), Gomes and Michaelides (2005), and Alan (2006). Jagannathan and Kocherlakota (1996) study the economic rationale behind the common financial advice that people should shift investments away from stocks and towards bonds as they age. Therefore they look at the *intensive margin* of the stock holdings over the life cycle. Cocco, Gomes and Maenhout (2005) also look at the intensive margin. They confirm Jagannathan and Kocherlakota's intuition by showing that labor income acts as a substitute for risk-free asset holdings in a realistically calibrated life cycle model with non-tradable labor income and borrowing constraints. The shape of the labor income profile over the life cycle thus induces the investor to reduce her optimal stock holding share when aging. Gomes and Michaelides (2005) do look at the stock market participation rate in a life cycle model with fixed entry cost, preference heterogeneity, and Epstein-Zin preference. However, since the model has not touched the correlation between the nonfinancial income and stock return and its change over the life cycle, it is not surprising that the model generates a flat stock market participation rate once people overcome the entry cost and fully step into the market[1], i.e., it can never replicate the decreasing participation rate when people get older. Alan (2006) focuses on the fixed stock market entry cost and structually estimates it in a life cycle intertemporal portfolio choice model. She finds that this cost is approximately equal to 2% of the permanent component of the annual labor income.

[1]See their Figure 4C panel A.

Our paper also extends the strand of literature on asset allocation under Kightian uncertainty such as Dow and Werlang (1992), Kogan and Wang (2002), Miao (2004), and Cao, Wang and Zhang (2005). This paper in spirit is closer to Miao (2004) and Cao, Wang and Zhang (2005). However, the significant difference between our paper and theirs is instead of modeling uncertainty on the mean of the risky asset payoff, we focus on the ambiguity on correlation between nonfinancial labor income and stock return. They only look at cross-section stock market participation rate, while we emphasize the life cycle pattern and especially the decreasing stock participation when aging. Under their framework, in order to explain why *older* people quit the stock market, we have to assume the ambiguity about the mean risky asset payoff becomes larger when people are aging, which we find not convincing. In fact, when we consider people can learn through experience, we would expect the opposite. However, ambiguity towards correlation between nonfinancial labor income and stock return through does increase when people face health shock. Therefore, it is a more natural candidate for explaining why older people quit the stock market.

In a recent paper, Yogo (2008) develops and calibrates a life cycle consumption and portfolio choice model with endogenous health expenditure. The goal is to explain the joint evolution of health status and the composition of wealth for retirees as observed in the data. His model has a standard expected utility function. And he only focuses on the effect of health shock on the stock share in wealth. Our work is significantly different from his paper in terms of deviation from expected utility framework and targeting on stock market participation.

The rest of the paper is organized as follows. Section 2 documents the stylized facts of life cycle stock market participation rate and explicitly show the participation decreases as people get older. Section 3 establishes it as a theoretical puzzle under standard expected utility framework. Section 4 presents our model which incorporates the health expenditure shock that only older people face and introduces ambiguity towards correlation between stock return and risky nonfinancial income (due to health shock) under a multi-prior utility framework. Section 5 concludes.

2 Facts

It has been noticed by Mankiw and Zeldes (1991), Haliassos and Bertaut (1995) and Vissing-Jørgensen (2002a), among others, that only a small fraction of the U.S. households hold stocks. In this paper, we look further into the *life-cycle* pattern of stock market participation. As Campbell (2006) and Ameriks and Zeldes (2004) point out, it's impossible to distinguish age, cohort, and time effect simultaneously when constructing age profiles. Therefore, we follow Campbell (2006) and assume that there is no cohort effect. This allows us to use SCF data surveyed in one year, which automatically controlling for time effect, for identifying the age effect. The following Figure 1 is constructed from the SCF data by looking at samples surveyed in 2004 with financial net worth more than $\$30,000$. There are more details about the SCF data and measurement of variables in the appendix. From this figure we see average stock market participation rate increases first until age 33, then it stablizes from age 33 to age 63. It begins to decrease sharply after age 63. From age 63 to 73,

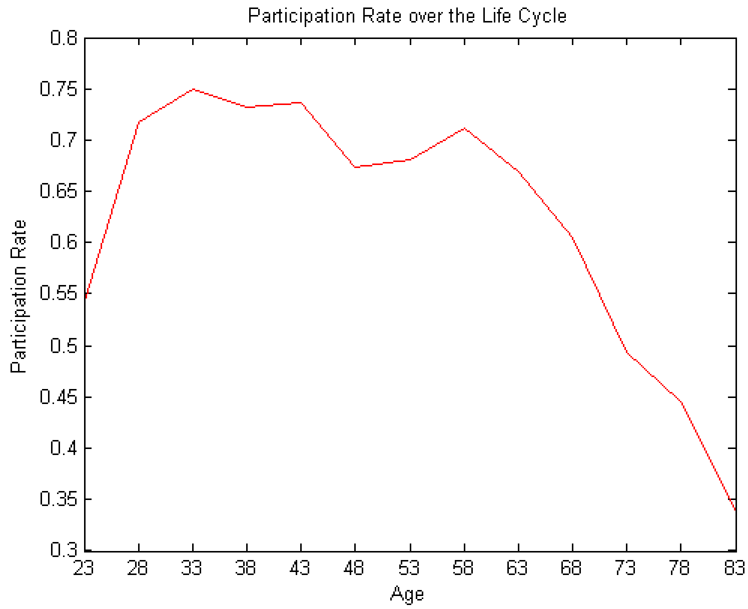

FIGURE 1: Stock market participation pattern over the life cycle

the stock market participation rate decreases by almost 20 percent. This decreasing trend for the older people is robust to definitions of stock investments and survey year. The pattern also holds by looking at different data set, such as PSID.

More importantly, the pattern even holds after controlling other demographic variables such as family size, race and education level, and financial factors such as income and wealth. Table 1 documents the results.

In column I of Table 1, we report logit regressions of stock market participation (i.e., extensive margin) on age and age squared controlling household income, wealth, and demographic characteristics. We find a negative coefficient -0.0003 (at 1% significance level) before age squared, which induces decreasing participation rate at older age.

Table 1. Equity Participation and Portfolio Share
The table reports demographic and other determinants of participation in stocks(public equity)(logit regressions, left panel) and of the stock portfolio share among participants(OLS regressions, right panel) for households in the 2004 SCF. Standard errors are reported underneath the coefficients in parentheses. Coefficients significant at the 10% level are denoted by *, at the 5% by **, and at the 1% level by ***.

	Public Equity		Portfolio Shares for Participants	
Dependent Variable	Coefficients I	Coefficients II	Coefficients III	Coefficients IV
Health Status		0.440		0.015
		(0.0558)***		(0.006)***
Any Pension		1.303		-0.019
		(0.051)***		(0.0056)***
Age	0.0075	-0.11	0.0015	0.00166
	(0.0079)	(0.008)	(0.001)	(0.001)
Age squared	-0.0003	-0.0002	$-1.79 * 10^{-6}$	-2.7$*10^{-6}$
	(0.000073)***	(0.000077)***	$(9.31 * 10^{-6})$	$(9.42 * 10^{-6})$
White	0.781	0.783	0.03	0.03
	(0.054)***	(0.556)***	(0.006)***	(0.006)***
High school diploma	0.657	0.608	0.067	0.064
	(0.081)***	(0.087)***	(0.011)***	(0.011)***
Some college	0.933	0.881	0.075	0.07
	(0.087)***	(0.0923)***	(0.011)***	(0.011)***
College diploma	1.302	1.243	0.11	0.104
	(0.084)***	(0.0899)***	(0.011)***	(0.011)***
Number of children	-0.187	-0.2043	-0.208	-0.021
	(0.022)***	(0.0238)***	(0.002)***	(0.019)***
Log(income)	0.2911	-0.705	-0.069	-0.054
	(0.6801)	(0.4800)	(0.047)	(0.0474)
Log(income) squared	0.017	0.051	0.002	0.0017
	(0.032)	(0.023)**	(0.002)	(0.002)***
Log(net worth)	0.252	-0.1598	-0.247	-0.244
	(0.1287)**	(0.098)	(0.023)***	(0.02)
Log(net worth) squared	0.014	0.034	0.01	0.01
	(0.0059)**	(0.0047)***	(0.0009)***	(0.0009)***
Sample size	4166	4166	2599	2599

The other fact deserves notice is that the major resource of non-asset income for older people is social security and pensions. The social security coverage rate for older people is over 94% in 2004 SCF survey. The social security benefits are annually adjusted according to CPI-W, and they're thus believed to be risk free. For the pension receivers, conditional on the fact that they're not stock market participants, the pension payment is risk free since the pension funds are not invested in stocks.

3 Why it's a puzzle

Two facts from the section above make the decreasing stock market participation among older people puzzling. First is that even for households with significant amount of financial assets, there is still a large fraction of non-participants. Second is that for older people, the non-asset income is risk free.

To establish it as a puzzle, we start with a simple two period portfolio choice model with standard expected utility function. The agents make consumption and portfolio choice to

$$\max_{B_0,S_0,C_0,C_1} u(C_0) + \beta \mathrm{E}_0[u(C_1)] \tag{1}$$

s.t. :

$$C_0 + B_0 + S_0 = W_0 \tag{2}$$

$$C_1 = Y + RB_0 + \widetilde{R}S_0 \tag{3}$$

For those people with positive financial asset, or those with $B_0 + S_0 > 0$, we first assume their optimal stock investment is zero, which implies $B_0 > 0$. It is easy to show that to reallocate a small positive amount Δ of assets from safe bonds to risky stocks will make them strictly better off because such move will bring change in utility by:

$$\beta \mathrm{E}_0 \left\{ u'(C_1)(\widetilde{R} - R) \right\} \cdot \Delta > 0 \tag{4}$$

The sign is determined by two facts. First, since we assume that stock investment is zero, the consumption is uncorrelated with stock return. Thus we have

$$\beta \mathrm{E}_0 \left\{ u'(C_1)(\widetilde{R} - R) \right\} \cdot \Delta = \beta \mathrm{E}_0 u'(C_1) \cdot \mathrm{E}_0(\widetilde{R} - R) \cdot \Delta$$

with standard utility function, we always have $\mathrm{E}_0 u'(C_1) > 0$. Second, the expected equity premium is positive which is found in data.

Even if we extend the above model to a multi-period model, as we have already known that for the second to the last period, the agent should keep investing in stocks. Besides, for any period the agent as a retired agent who is receiving non-risky social security income can adopt the invest-and-stay strategy to beat non-participation strategy.

The above results will overturn the hypothesis for non-participation. In other words, people should always stay in stock market. This argument establishes the decreasing stock participation rate for older people as a puzzle.

The puzzle is robust in many ways. First, it's actually independent of the degree of risk aversion since risk aversion coefficient never enters into Equation (4). This observation makes the stock participation puzzle even "deeper" than the equity premium puzzle. High degree of risk aversion might reconcile equity premium puzzle, as argued by Kandel and Stambaugh(1991). However, the stock participation puzzle for older people can never be resolved by simply increasing the magnitude of risk aversion.

Neither changing discount rate nor introducing survival probability can solve the puzzle. Since they both can not change the sign of marginal utility in equation (4).

The puzzle still exists if bequest motive is introduced. With bequest motive, agents have a stronger incentive to save. Given the positive equity premium, they should first consider allocate their extra savings into stocks.

Similarly, neither "catching up with the Joneses" or habit formation story can solve this participation puzzle. Because with such modifications, the correlation between marginal utility is still uncorrelated with stock return for non-participants, and so by transforming some safe assets into stocks can always make the agents better off. Other modifications on preferences such as Epstein-Zin utility will not work due to the same reason.

We argue that market frictions such as borrowing constraints or short sell constraints can not solve the puzzle either. The puzzle is established upon the fact that quitting the stock market happens to those with positive amount of financial assets (see Figure 1). This means that they are far away from binding borrowing constraints. Due to the positive equity premium, short sell constraint should never bind and it thus can't solve the puzzle. Fees and taxes can't change the sign of equity premium, so they are not powerful enough to prevent people keep participating. One time entry cost can't work here either, since it can't explain why those people who already overcomed the fixed entry cost quit stock market when they get older. Other friction such as minimum investment requirement, which is usually around $500 for most funds nowadys, requires risk aversion extremely high in order to make people quit stock market. Haliassos and Bertaut(1995) confirms this argument.

The intuitions given above can also be verified by Gomes and Michaelides(2005), where realistic income process is estimated from PSID and put into a life cycle portfolio choice model with fixed entry cost. It predicts that stock market participation rate keeps 100% for all mid-aged and older people.

4 The Model

To find a way out to solve the puzzle that is established by data, we first look back into data. Rosen and Wu(2004) find that health status is a very significant factor to predict stock market participation. The table below (taken from their paper) shows the striking difference among people with different health status. Healthier people tend to more likely participate in the stock market. And according to Rosen and Wu (2004), this pattern still holds after controlling other factors such as risk aversion, planning horizon, bequest motive, and health insurance. Motivated by Rosen and Wu (2004), we use 2004 SCF data to run logit regressions on stock market participation on self-reported health status, controlling household income, wealth,

	single	married
Sick	8.2%	12.2%
Healthy	25.1%	38.5%

and demographic characteristics. Our finding is reported in column II of Table 1. Our health status here is a discrete variable has two values. 0 represents "non-healthy," while 1 represents for "healthy." 0 corresponds to 3-4 self-reported health status as in original SCF data. 1 corresponds to 1-2 self-reported health status in SCF. We find health status is a significantly positive predictor of stock participation. This is also true for stock share in portfolio as in column IV.

4.1 Two Period Model

We consider an agent's consumption, saving and investment decision in a two-period model. The agent can invest in both risk free bonds and stocks. In period zero, the agent holds initial wealth W_0, a pool of his bond holding, stocks holding and labor income. He decides how much to consume, to invest in bonds and stocks. In period 1, the agent receives risky labor income $\widetilde{Y}$ and returns from his investment. The gross return to the stocks is $\widetilde{R}$, that for bonds is R. Both $\widetilde{R}$ and $\widetilde{Y}$ are random variables defined on probability space $(\Omega, \mathcal{F}, P)$.

The agent has a set of priors $\mathcal{P}$ that over $(\Omega, \mathcal{F})$. The implicit assumption for multiple priors is that the agent is not able to form a single prior for the joint performance of his de facto income and stock return. We assume that the ambiguity is only towards the correlation between income and stock return. [1] We assume the reference distribution P for joint performance of stock and income is:

$$(\widetilde{R},\widetilde{Y}) \overset{\mathrm{P}}{\sim} \mathrm{N}\left(\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix},\begin{pmatrix}\sigma_1^2 & \sigma_{12}\\ \sigma_{12} & \sigma_2^2\end{pmatrix}\right) \tag{5}$$

And the set of priors is

$$\mathbb{P}(\mathrm{P}) = \left\{\ \mathrm{Q}|(\widetilde{R},\widetilde{Y}) \overset{\mathrm{Q}}{\sim} \mathrm{N}\left(\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix},\begin{pmatrix}\sigma_1^2 & \sigma_{12}+v\\ \sigma_{12}+v & \sigma_2^2\end{pmatrix}\right) : v \in [\underline{v},\overline{v}]\ \right\} \tag{6}$$

And we assume that $0 \in [\underline{v},\overline{v}]$.

The agent has multi-prior utility as in Gilboa and Schmeidler (1989). We assume the utility function is CARA. Thus the objective function is

$$u(C_0) + \beta \min_{Q\in\mathbb{P}} [u(C_1)] \tag{7}$$

where $u(c) = -\frac{1}{\theta}\exp(-\theta c)$, where $\theta > 0$. Thus we can summarize the agent's

[1] Gilboa and Schmeidler's (1989) work does not impose any restrictions on structure of $\mathcal{P}$. Some work, including Kogan and Wang (2002), Cao, Wang and Zhang (2005), and Miao (2004), among others, use entropy criterion to define $\mathcal{P}$, which can deliver analytical and intuitive results. Their work assume that the ambiguity is only towards the mean of stock return, we will show later that this way of modeling can't deliver the pattern of quitting from stock market as we observe from data.

problem in two-period setting as follows:

$$\max_{B_0,S_0} \quad u(C_0) + \beta \min_{Q\in\mathbb{P}} [u(C_1)] \tag{8}$$

s.t. :

$$C_0 + B_0 + S_0 = W_0 \tag{9}$$

$$C_1 = \widetilde{Y} + RB_0 + \widetilde{R}S_0 \tag{10}$$

To solve the model, we need first solve the inside minimization problem, and this is equivalent to pick a proper value of disturbance on correlation. Under the assumption that a generic distribution is normal and that utility function is CARA, we can rewrite the objective function depending on the sign of stock holdings and then solve it accordingly.

Case 1: if $S_0 > 0$,the objective function becomes:

$$\max_{B_0,S_0} \quad -\frac{1}{\theta}\exp\left[-\theta(W_0 - B_0 - S_0)\right] - \frac{\beta}{\theta}\exp\left[-\theta RB_0 - \theta S_0\mu_1 - \theta\mu_2 + \frac{\theta^2}{2}\left(S_0^2\sigma_1^2 + 2S_0\left(\sigma_{12} + \overline{v}\right) + \sigma_2^2\right)\right] \tag{11}$$

Solving this for S_0 we can get

$$S_0 = \frac{\mu_1 - R - \theta(\sigma_{12} + \overline{v})}{\theta\sigma_1^2} \tag{12}$$

where $\overline{v} < \frac{\mu_1 - R}{\theta} - \sigma_{12}$.

The intuition is that, due to the ambiguity towards the correlation, the equity premium is discounted by risk coefficient times the correlation. And the investor longs in stocks only when the discounted equity premium is still positive under the "worst" scenario, where "worst" means when stocks and income are most like each other. If the stocks and income are so much like each other that at some point, the agent will quit the stock market to avoid risks.

Similarly, under case 2, if $S_0 < 0$, we get

$$S_0 = \frac{\mu_1 - R - \theta(\sigma_{12} + \underline{v})}{\theta \sigma_1^2} \tag{13}$$

where $\underline{v} > \frac{\mu_1 - R}{\theta} - \sigma_{12}$.

Similar intuition applies here. The investor shorts stocks only when in the "best" scenario the discounted equity premium is still negative. And here "best" scenario is when stocks and income are very dislike each other. If stocks and income are too dislike each other, the investor would not have incentive to hedge against his income uncertainty by short selling stocks.

The last and most important case is when agent is not participating in stocks. We have the necessary condition for non-participation:

$$\underline{v} \leq \frac{\mu_1 - R}{\theta} - \sigma_{12} \leq \overline{v} \tag{14}$$

The agent is not longing since he's worried that his income will be too much like a stock. His nightmare is that the stock return will be low when his non-asset income is also low. Meanwhile, due to the ambiguity, the agent is also worried that his income will be too dislike a stock. He is concerned with the following scenario: he shorts in stocks, and his income hits a low shock. However, at the same time, the stock return is high, meaning he will be hurt by buying stocks with higher prices to pay back his short selling. The multiple prior utility captures these two direction worries at the same time, and therefore deliver the non-participation.

This two period model offers enough intuitions about why we need ambiguity towards the correlation to solve the puzzle. In the following, we will show other alternative specifications do not work.

No ambiguity Assume for now there is no ambiguity in the model. We have a single prior of joint distribution between stock return and non-asset income then all agents share the same single belief about the joint distribution of stock return and non-asset income as in equation (5). Then it is easy to show that the stock holding is

$$S_0 = \frac{\mu_1 - R - \theta\sigma_{12}}{\theta\sigma_1^2} \tag{15}$$

Given expected equity premium is positive, we have $S_0 > 0$. Nobody will ever quit the stock market.

Ambiguity towards the mean of non-asset income Alternatively, if we assume that the ambiguity is towards the mean of the last period non-asset income

$\widetilde{Y}$, the set of priors changes to

$$\mathbb{P}(\mathrm{P}) = \{ \ \mathrm{Q}|(\widetilde{R}, \widetilde{Y}) \overset{\mathrm{Q}}{\sim} \mathrm{N} \left(\begin{pmatrix} \mu_1 \\ \mu_2 - v \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right) : v \in [\underline{v}, \overline{v}] \ \} \, .$$

We can show that it would change the consumption-savings (bond holding) decision. However, it would not change the stock participation decision since the mean of non-asset income μ_2 never enters into the equation of stocking holding as in (13).

Ambiguity towards the mean of stock return Last, if we assume that the ambiguity is towards the lmean of the last period stock return $\widetilde{R}$, the set of priors now is as follows

$$\mathbb{P}(\mathrm{P}) = \{ \ \mathrm{Q}|(\widetilde{R}, \widetilde{Y}) \overset{\mathrm{Q}}{\sim} \mathrm{N} \left(\begin{pmatrix} \mu_1 - v \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right) : v \in [\underline{v}, \overline{v}] \ \} \, .$$

In this case, the optimal stock holding is given by

$$S_0 = \begin{cases} \frac{(\mu_1 - \overline{v}) - R - \theta\sigma_{12}}{\theta\sigma_1^2} & \overline{v} < \mu_1 - R - \theta\sigma_{12} \\ 0 & \underline{v} \leq \mu_1 - R - \theta\sigma_{12} \leq \overline{v} \\ \frac{(\mu_1 - \underline{v}) - R - \theta\sigma_{12}}{\theta\sigma_1^2} & \underline{v} > \mu_1 - R - \theta\sigma_{12} \end{cases}$$

We do generate non-participation region depending on the dispersion of ambiguity. However, in order to explain why *older* people quit the stock market, which is the puzzle we are interested, we have to assume the ambiguity about the mean of risky stock return becomes larger when people are aging. We find it is not convincing.

In fact, when we consider people can learn through experience, we would expect the opposite. Besides, it says nothing about the data fact that health status is a significant predictor of stock market participation.

4.2 A Simple Dynamic Model

To qualitatively capture the stock market participation pattern over the life cycle, we extend two period model to a three period model. The three periods are 0,1,2.

We assume an agent has income profile (W, aW, bW) over three periods. Parameters a, b are designed to approximate real life income profile, where $a > 1$ can approximate the peak of mid-aged income and bW can capture the social security benefits, which depends on agent's previous income. The income profile is certain. The agent faces the health expenditure shock at the last period. The expenditure is a random variable denoted by $\widetilde{H}$. In each period, the agent decides how much to consume and to invest in bonds and stocks. The returns on stocks are uncertain. And there is ambiguity towards the joint distribution on last period stock return and health expenditure shock. We assume that the above joint distribution is independent of first period stock return. These assumptions guarantee the regularity condition hold. (See Epstein and Schneider (2003)[2]). Henceforth, we can use backward induction to solve the maximization problem.

[2]Here the one-step-ahead belief as defined in Epstein and Schneider (2003) is a collection of joint distributions on stock return and health shock in period 2. This collection is independent of the realization of period 1 stock return due to the assumption of independence, which, in turn, guarantees the rectangularity condition satisfied. We thus have the legitimacy to use the one-stop-ahead belief rather than belief at time 0 when we define the utility function.

The three period problem can be summarized as[3]:

$$\max_{B_0,S_0,B_1,S_1} \quad u(C_0) + \beta E_1\left[u(C_1)\right] + \beta^2 \min_{Q\in\mathbb{P}}\left[u(C_2)\right] \tag{16}$$

s.t. :

$$C_0 + B_0 + S_0 = W \tag{17}$$

$$C_1 + B_1 + S_1 = aW + RB_0 + \widetilde{R_1}S_0 \tag{18}$$

$$C_2 = (bW - \widetilde{H}) + RB_1 + \widetilde{R_2}S_1 \tag{19}$$

Since we use backward induction to solve this problem, we can use the results from the two period model and determine the stock holding at time 1 as follows:

$$S_1 = \begin{cases} \frac{\mu_1 - R - \theta(\sigma_{12}+\overline{v})}{\theta\sigma_1^2} & \overline{v} < \frac{\mu_1 - R}{\theta} - \sigma_{12} \\ 0 & \underline{v} \le \frac{\mu_1 - R}{\theta} - \sigma_{12} \le \overline{v} \\ \frac{\mu_1 - R - \theta(\sigma_{12}+\underline{v})}{\theta\sigma_1^2} & \underline{v} > \frac{\mu_1 - R}{\theta} - \sigma_{12} \end{cases} \tag{20}$$

where non-asset income $\widetilde{Y}$ is equal to $bW - \widetilde{H}$. And we assume that the set of joint distributions on $\widetilde{Y}$ and $\widetilde{R}_2$ is similar to that in definition (2).

The bond holding, in case when $S_1 > 0$, is:

$$B_1 = \frac{\ln \beta R}{\theta(1+R)} + \frac{W_0 - \mu_2 - (1+\mu_1)S_1}{1+R} + \frac{1}{1+R}\frac{\theta\sigma_2^2}{2} + \frac{1}{1+R}\frac{\theta S_1^2\sigma_1^2}{2} + \frac{\theta S_1(\sigma_{12}+\overline{v})}{1+R} \tag{21}$$

where the first term is the savings caused by impatience. It's positive if interest rate

[3] The utility function is still CARA.

dominates time discount factor ($\beta R > 1$). The second term is the difference between tomorrow's expected income and today's wealth. Here $W_0 = aW + RB_0 + R_1 S_0$ is the total wealth received at the beginning of period 2, where R_1 is the realization of period 1 stock return. The third term is the precautionary savings due to uncertainty in income. Similarly, the forth term is the precautionary savings caused by uncertainty in stock return. The last term is precaution saving for the co-movement between stock and income.

For the case when $S_1 < 0$, the B_1 formula will be modified accordingly to the last term, which is changed to $\frac{\theta S_1(\sigma_{12}+\underline{v})}{1+R}$. When $S_1 = 0$, the bond holding is:

$$B_1 = \frac{\ln \beta R}{\theta(1+R)} + \frac{W_0 - \mu_2}{1+R} + \frac{1}{1+R}\frac{\theta\sigma_2^2}{2} \tag{22}$$

obviously, it is absent of the two precautionary savings terms due to stock holding uncertainty and ambiguity towards correlation between stock return and de facto income.

Using backward induction, in period 0, it's easy to show that everyone holds stock as follows

$$S_0 = \frac{1+R}{R}\frac{\mu_1 - R}{\theta\sigma_1^2} \tag{23}$$

In fact, the time 0 stock holding is bigger than zero for every agent, therefore, the model predicts that the stock market participation rate in period 0 is 1, however, due to the existence of ambiguity, someone will quit the stock market in period 1.

Hence we deliver the decreasing stock market participation for older people in this model. It's also worth pointing out that S_0 is larger than S_1, therefore we generate the decreasing stock share for older people over the life cycle.

5 Who Quit and Who Stay in Stock Market

From the discussion above, we found that in this model the key component to determine non-participation is the range of band on ambiguity towards the correlation between stock return and non-asset income.

The non-participation condition in (14) is that:

$$\underline{v} \leq \frac{\mu_1 - R}{\theta} - \sigma_{12} \leq \overline{v}$$

where $\sigma_{12} = \mathrm{Cov}_{\mathrm{P}}(\widetilde{Y}, \widetilde{R})$. Here P is the reference distribution as defined in (5). We also have $\widetilde{Y} = bW - \widetilde{H}$. Where $\widetilde{H}$ is the random health expenditure. We can rewrite $\widetilde{H} = W\widetilde{h}$, i.e., $\widetilde{h}$ is a share of health expenditure over initial period labor income W. Henceforth, the correlation for reference distribution can be rewritten as:

$$\sigma_{12} = W[E(\widetilde{h})E(\widetilde{R}) - E(\widetilde{h}\widetilde{R})]$$

and the non-participation condition can be rewritten as:

$$\underline{v} \leq \frac{\mu_1 - R}{\theta} + W[E(\widetilde{h}\widetilde{R}) - E(\widetilde{h})E(\widetilde{R})] \leq \overline{v} \tag{24}$$

First we assume people have same knowledge of μ and R. When people share same parameters of risk aversion, ambiguity level $E(\widetilde{h}\widetilde{R}) - E(\widetilde{h})E(\widetilde{R})$, the higher is W, the less likely that people will stop participating. If $E(\widetilde{h}\widetilde{R}) - E(\widetilde{h})E(\widetilde{R}) > 0$, the agent will likely to long if W is big enough. And they will short when W is big enough and $E(\widetilde{h}\widetilde{R}) - E(\widetilde{h})E(\widetilde{R}) < 0$.

If we assume people have same parameters except for ambiguity level, then those with smaller band of ambiguity will less likely stop participating.

Similarly, if people have same parameters as others except for risk aversion θ, then the less risk averse the agent is, the more likely he will long stocks.

6 Conclusion

This paper documents a life cycle pattern of stock market participation rate. We find that the stock market participation rate decreases dramatically after age 60. Why do older people quit the stock market?

We provide an answer based on the observation that older people face much stronger health risk than young and the empirical evidence that shows the health risk is indeed a powerful predictor of stock participation (Rosen and Wu 2004). In our model, older people face health expenditure shock. In addition, they are uncertain about the correlation between stock return and de facto nonfinancial income which is negatively affected by their health expenditure. Under a multi-prior utility framework, we show that there exists a range of ambiguity level towards this correlation over which investors neither long or short sell a stock. Heterogeneity among older people on the ambiguity towards the correlation between risky stock return and uncertain health expenditure thus guarantees some people will fall into this range. Therefore they will choose not to participate in the stock market.

Appendix

Appendix A: Data

The Survey of Consumer Finances (SCF) data is a triennial survey data designed to provide detailed information on the finances of the U.S. households. It is sponsored by the Board of governors of the Federal Reserve System in cooperation with Internal Revenue Service of the Department of Treasury. And since 1992, data have been collected by the National Organization for Research at the University of Chicago (NORC).

The SCF data have a good coverage of US households. Thanks to a dual-frame sample design, the SCF data has one subset with a good coverage of characteristics such as age, education, income and wealth level (e.g., there are 3,007 cases the this set in 2004 survey); while in the other subset it oversamples relatively wealthy households drawn from a list of records offered by IRS (there are 1,515 cases in 2004 survey). By doing so it helps to enhance accuracy with respect to asset allocation, since wealthy people tend to more likely to hold and to hold more financial assets. Proper weights are then applied to both samples in order to make estimations right for the whole population.

The SCF data covers all categories of household wealth. For example, as to the stock holding, it asks in great details how and how much households invest in stocks, either directly, or through mutual funds such as stock mutual funds and combination mutual funds, or through other assets such as annuities, trust or thrift type retirement account.

Besides, SCF has applied computer assisted interviewing program to enhance quality of survey data. There usually exists high frequency of "don't know" responses to questions asking about value of assets. The computer program, however, uses a host of ways to extract information and to avoid "don't know" responses. For example, it offers range cards to those who are uncomfortable saying exact number to specify range. And it also uses series of questions in a decision tree to clarify the range more specifically.

The above advantages make SCF the best data set to study household finances.

Throughout the paper, the variable age refers to that of the head of a primary economic unit (PEU). Where PEU is the household unit in the SCF data and it consists of a core single individual or couple in a household and all others who are financially dependent on that individual or couple.

Stock investment has three different measures in SCF data set. We use the one with the broadest definitions, while the participation pattern over the life cycle is robust with respect to the other two definitions. The stock investment here incorporates stocks directly held by household, investments in stock mutual funds, investments in combination mutual funds (half of the value counted as stocks), assets in IRA or Keogh accounts that are invested in stocks, and assets in annuities, trusts, and managed investment accountants that are invested in stocks.

In SCF, net worth refers to difference between all assets, including financial and non-financial ones, and debts. We also construct other definitions of assets for robustness check. For example, following Heaton and Lucas (2000), we define total financial net worth, total financial assets, and liquid assets to examine the stock market participation pattern over the life cycle.

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